CHAPTER 6 – PRODUCTION

Key Concepts and Topics

- The Technology of Production
- Production with One Variable Input (Labor)
- Isoquants
- Production with Two Variable Inputs
- Returns to Scale

Introduction

- Theory of the firm
  - Explains how a firm makes *cost-minimizing* production decisions and how its *costs vary with output*. Costs of production change with changes in both *input prices* and *level of output*

Production Decisions of a Firm

- Production decisions of a firm can be broken down into three steps:
  1. Production Technology
  2. Cost Constraints
  3. Input Choices

- Production Technology
  - Describe how *inputs* can be transformed into *outputs*
    - ♦ *Inputs*: land, labor, capital and raw materials
    - ♦ *Outputs*: cars, desks, books, etc.
  - Firms can produce different amounts of outputs using *different combinations of inputs*

- Cost Constraints
  - Firms must consider *prices* of labor, capital and other inputs
  - Firms want to *minimize* total production costs partly determined by *input prices*
  - Firms must be concerned about *costs of production*

- Input Choices
  - Given input prices and production technology, the firm must choose *how much of each input* to use in producing output

- Given prices of different inputs, the firm may choose different combinations of inputs *to minimize costs*
  - If labor is *cheap*, may choose to produce with *more* labor and *less* capital

- If a firm's costs of production are at its *minimization*, we can study
  - How *total costs of production* varies with *output*
  - How does the firm choose *the quantity* to *maximize* its *profits*

- We can represent the firm's production technology in the form of a *production function*

The Technology of Production

- Production Function
  - Describes the *maximum output* (q) that a firm can produce for *every specified combination of inputs* at a given state of technology
  - For simplicity, we will consider only *labor* (L) and *capital* (K)
  - Shows what is *technically feasible* when the firm operates efficiently

- The production function for two inputs: $\boldsymbol{q = F(K,L)}$
  - Output (q) is a function of capital (K) and labor (L)
  - Both inputs and output are *flows*
  - The production function is true for a given *technology*
  - If *technology* increases, *more* output can be produced for a given level of inputs

- Short Run versus Long Run
  - It takes time for a firm to *adjust production* from one set of inputs to another
  - Firms must consider not only what *inputs* can be varied but over what *period of time* that can occur
  - We must distinguish between *long run* and *short run*
  - Short run and long run are not *time specific*

- Short Run
  - Period of time in which *at least* one of the inputs is *fixed*
  - For example, when K is *fixed*, firms vary the *intensity* with which they utilize a given plant and machinery

- Long-run
  - Amount of time needed to make *all* production inputs *variable*

Production with One Variable Input (Labor)

- Assume K is *fixed* and L is *variable* (i.e., a *short run* analysis)
  - Output can only be increased by increasing L

Amount of Labor (L)	Amount of Capital (K)	Total Output (q)
0	10	0
1	10	10
2	10	30
3	10	60
4	10	80
5	10	95
6	10	108
7	10	112
8	10	112
9	10	108
10	10	100

- Observations: *How output changes as the amount of labor is changed?*
  - When labor is *zero*, output is *zero*
  - With additional workers, output increases up to *8* units of labor
  - Beyond *8* units of labor, output *declines*
    - ♦ Increasing labor can make better use of *existing capital initially*
    - ♦ After a point, more labor is *not useful* and can be *counterproductive*

- Firms make decisions based on the *benefits* and *costs* of production
  - Sometimes useful to look at *benefits* and *costs* on an *incremental* basis
    - ♦ How much more can be produced with an additional unit of an input
  - Sometimes useful to make comparison on an *average* basis

- Average Product (*AP*)
  - Output *per unit* of a particular input

- Average Product of Labor (AP_L)
  - Measures the *productivity* of a firm's labor in terms of how much, *on average*, each worker can produce

$$AP_L = \frac{Output}{LaborInput} = \frac{q}{L}$$

- Marginal Product (MP)
  - *Additional* output produced by one *additional* unit of a *particular* input, holding all other inputs *constant*
  - In other words, the *extra* output obtained from the *last* unit of input added

- Marginal Product of Labor (MP_L)
  - Measures the *additional* output produced by one *additional* unit of labor, holding all other inputs *constant*

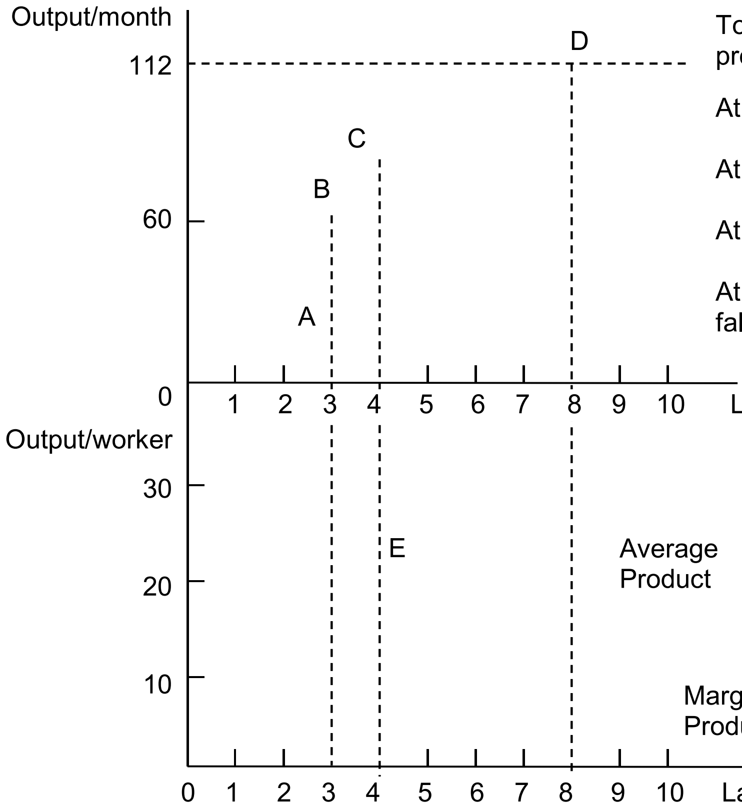
$$MP_L = \frac{\Delta Output}{\Delta LaborInput} = \frac{\Delta q}{\Delta L}$$

- Example: Production with one variable input (labor)

Amount of Labor (*L*)	Amount of Capital (*K*)	Total Output (*q*)	Average Product (*q/L*)	Marginal Product ($\Delta q/\Delta L$)
0	10	0	–	–
1	10	10		
2	10	30		
3	10	60		
4	10	80		
5	10	95		
6	10	108		
7	10	112		
8	**10**	**112**		
9	10	108		
10	10	100		

- We can graph the information to show
  - How output varies with changes in labor
    - Output is *maximized* at *112* units
  - Average and Marginal Products of Labor
    - MP_L is *positive* as long as total output is *increasing*
    - MP_L *crosses* AP_L at its *maximum*

Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.



Total Product Curve shows the output produced for different amount of labor input

At A, MP = slope of the tangent = 40/2= 20
> AP = 15
At B, AP = slope of the line 0B = 60/3 = 20
< MP = 30
At C, AP = slope of the line 0C = 80/4 = 20
= MP = 20
At D, output is maximized; total product falls thereafter. AP = 112/8 = 14
> MP = 0 = slope of the tangent

AP at its maximum when labor is 4

At E, MP = AP

Left of E, MP > AP and AP ↑

Right of E, MP < AP and AP ↓

– Positive as long as output ↑
– Negative when output ↓

Negative MP for the additional labor who slows production and decreases output

- Average Product and Marginal Product
  - When *MP* > *AP*, *AP* is *increasing*
  - When *MP* < *AP*, *AP* is *decreasing*
  - When *MP* = *0*, total product (output) is at its *maximum*
  - *MP crosses AP* at its *maximum*

- Total Product, Average Product and Marginal Product Curves
  - *Average product* is the *slope* of the line from the *origin* to any corresponding point on the total product curve
    - At point B, *AP* = 60/3 = 20, which is the same as the *slope* of the line from the *origin* to point B on the total product curve

  - *Marginal product* is the *slope* of the line *tangent* to any corresponding point on the total product curve
    - At point A, *MP* = 20, which is the same as the *slope* of the *tangent* line to the total product curve at point A
    - At point D, the *maximum* output, *MP* = 0, same as the *slope* of the *tangent* line
    - Beyond D, *MP* < 0

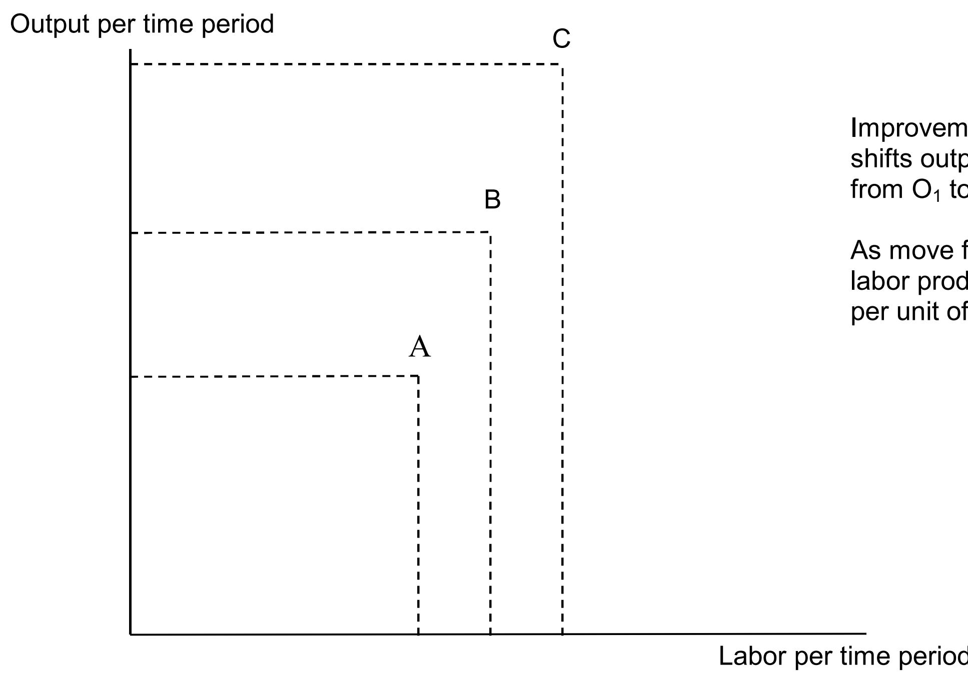
Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.

Law of Diminishing Marginal Returns

- As the use of an input *increases*, with other inputs and production technology *fixed* (i.e., in the *short run*), the resulting additions to output will eventually *decrease*
  - Limitations on the use of other *fixed* inputs
    - When L is *small* and K is *fixed*, *q increases* considerably since workers can begin to *specialize* and MP_L *increases*
    - When L is *large*, some workers become *less efficient* and MP_L *decreases* (because each additional L has *less* K to work with)
  - Not from declines in *quality* of the variable input (all Ls are of equal *quality*)
  - *Declining* marginal product, *NOT* necessarily a *negative* return
    - Additional output can be *declining* while total output is *increasing*

- Assumes a constant technology
  - Changes in technology will cause *shifts* in the total product curve
  - *More* output can be produced with *same* inputs
  - Labor productivity can *increase* if there are improvements in *technology*, even though any given production process exhibits *diminishing returns* to labor

- The Effect of Technological Improvement



Improvements in technology shifts output curve upward from O_1 to O_2 to O_3

As move from A to B to C labor productivity (output per unit of labor) increases

Labor Productivity (*LP*)

- The *average product of labor* for an entire industry or the economy as a whole
  - Links *macro* and *microeconomics*
  - Can provide useful comparisons across *time* and across *industries*

- Labor productivity and standard of living
  - Consumption can increase *only if* productivity increases
    - The aggregate value of goods and services produced by an economy (e.g., GDP) in any particular year is *equal to* the payments made to all factors of production and profit of firms
    - Consumers are the ultimate *recipients* of such factor payments in the form of wages, salaries, dividends, or interest
    - Hence, consumers in the *aggregate* can increase their rate of consumption (i.e., *improve their standard of living*) in the long run by *increasing* their productivity

- Causes of productivity growth
  - *Growth in stock of capital* – total amount of *capital* available for production ($\uparrow K \Rightarrow$ more and better machinery, each worker can produce more output for each hour worked)
  - *Technological change* – development of new *technologies* that allow factors of production to be used more *efficiently*

Production with Two Variable Inputs

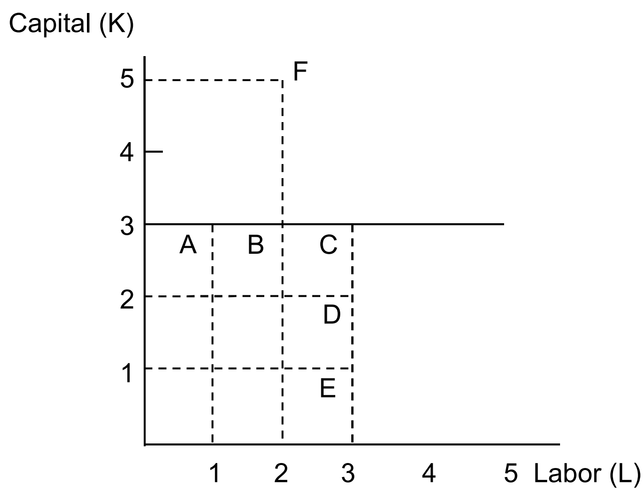
- In the long run, firm can produce output by combining *different* amounts of labor (*L*) and capital (*K*)
- It is important for firms to know the *incremental* output of each factor, while holding the other inputs *constant*, so that the information can be used to choose the *cost-minimizing* combination of inputs to produce a given output

Production with Two Variable Inputs

	Labor Input				
Capital Input	**1**	**2**	**3**	**4**	**5**
1	20	40	55	65	75
2	40	60	75	85	90
3	55	75	90	100	105
4	65	85	100	110	115
5	75	90	105	115	120

Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.

- We can graph the information using *isoquants*



- Isoquants show all combinations of L and K that together produce the output $q_1 = 55$ units, $q_2 = 75$ units, $q_3 = 90$ units
- Above and to the right of q_1 obtaining a higher level of output because more *L* and *K* are used
- Increasing *L* holding *K* constant (A, B, C) or increasing *K* holding *L* constant (E, D, C), q increases at a decreasing rate (0, 55, 20, 15) due to DMR

- *Isoquant* (*Equal Product Curve*) – A curve that shows all *possible* combinations of inputs that yield the *same* output, given a production technology

- *Isoquant Map* – A set of *isoquants* that describes a firm's *production function*. Each *isoquant* corresponds to a different level of *output*. Curves that lie farther northeast from the origin represent *higher* level of output

- Input Flexibility
  - *Isoquants* show the *flexibility* that firms have when making production decisions – *substituting* one input for another to produce a *specific* output
  - Knowing such *flexibility* is essential for choosing the input combination that *minimize costs* and *maximize profit*

- Diminishing Marginal Returns (*DMR*)
  - In the *short run*, both *K* and *L* can exhibit *DMR* because *adding* one factor while holding the other factor *constant* will eventually lead to lower and lower *incremental* output. The *shape* of *isoquants* demonstrates *DMR* – *steeper* as more *K* is used and *flatter* as more *L* is used

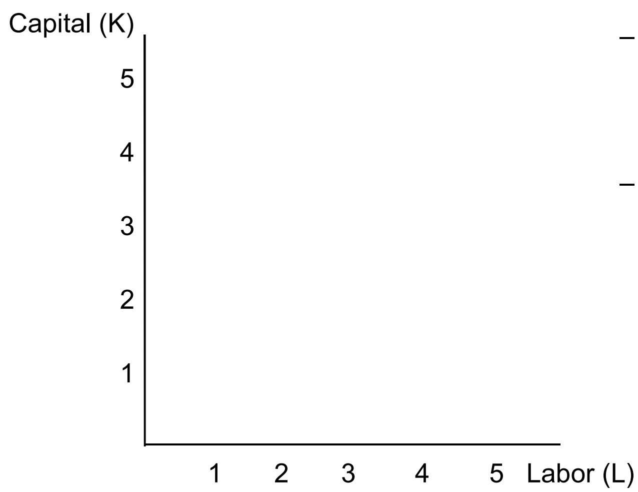
  - Diminishing returns to labor
    - ♦ Holding *K* constant at 3 and increasing *L* from 0 to 1 to 2 to 3
    - ♦ *q* increases at a *decreasing* rate (0, 55, 20, 15) due to *DMR* from labor

  - Diminishing returns to capital
    - ♦ Holding *L* constant at 3 and increasing *K* from 0 to 1 to 2 to 3
    - ♦ *q* increases at a *decreasing* rate (0, 55, 20, 15) due to *DMR* from capital

- Marginal Rate of Technical Substitution (*MRTS*)
  - Shows the amount by which one input can be *reduced* when *one extra* unit of another input is used, so that output remains *constant*
  - Measured by the *negative* slope of the *isoquants* at a *particular* point

$$MRTS_{LK} = \frac{ChangeInCapitalInput}{ChangeInLaborInput} = -\frac{\Delta K}{\Delta L}$$

  - The *convexity* of an isoquants indicates that *MRTS* falls as we move down the curve



- The slope of the isoquants at any point measures the *MRTS* – the ability of the firm to replace *K* with *L*, holding *q* constant
- *MRTS* falls from 2 to 1 to 2/3 to 1/3 as move down the isoquant, which indicates that as more and more *L* is used, *L* becomes relatively less productive and while less and less *K* is used, *K* becomes relatively more productive. Therefore, less *K* is required to keep *q* fixed, and the isoquants becomes flatter

- As increase labor to replace capital
  - Labor becomes *relatively less productive*
  - Capital becomes *relatively more productive*
  - Need *less* capital to keep output *constant*
  - *Isoquant* becomes *flatter*

- *MRTS* and Isoquants
  - We assume there is *diminishing MRTS*
    - *Increasing* labor in one unit increments from *1* to *5* results in a *decreasing MRTS* from *2* to *1/3*
    - Productivity of any one input is *limited*
    - Production needs a *balanced* mix of both inputs
  - *Diminishing MRTS* occurs because of *diminishing returns* and implies *isoquants* are *convex*

Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7^{th} Ed., Pearson Prentice Hall, Chapter 6.

- *MRTS* and Marginal Products
  - The increase in q from the increase in L = Amount of L increased times the *marginal productivity of labor* (i.e., the *additional* output per unit of *additional labor*)

$$\Delta q = (MP_L)(\Delta L)$$

  - Similarly, the decrease in q from the decrease in K = Reduction of K times the *marginal product of capital*

$$\Delta q = (MP_K)(\Delta K)$$

  - If q is holding *constant*, the net effect of increasing L and decreasing K must be *zero*

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

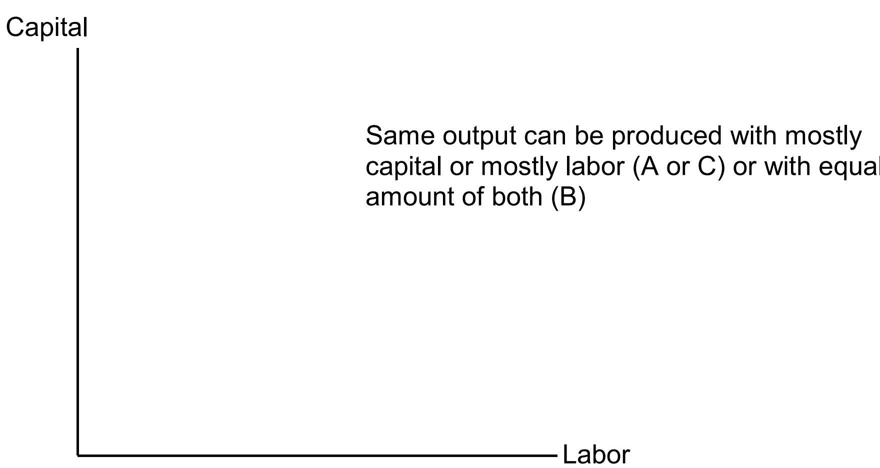
$$(MP_L)(\Delta L) = -(MP_K)(\Delta K)$$

$$\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = MRTS$$

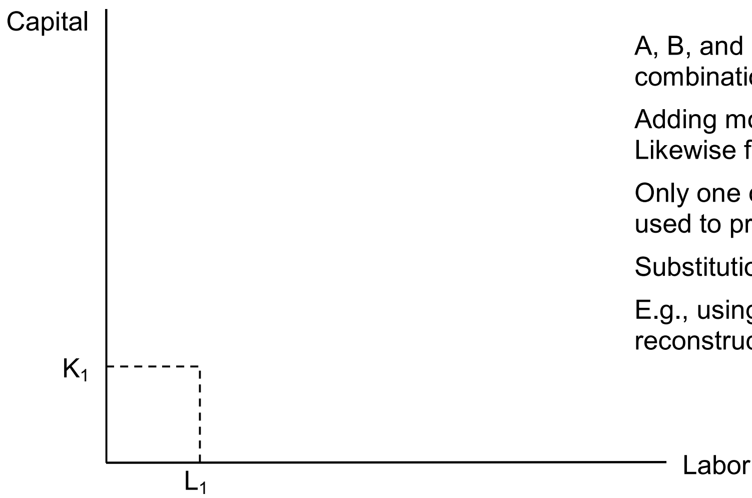
  - *The MRTS between two inputs is equal to the ratio of the marginal products of the inputs*

Production Functions: Two Special Cases

- Perfect Substitutes
  - The isoquants are *straight lines*
  - *MRTS* is *constant* at all points on isoquants
  - Any *substitution* between the inputs in the production process is *possible*
    - ♦ Same q can be produced with a lot of K or a lot of L or a *balanced mix*



- Perfect Complements
  - *Fixed proportions* production function
  - The isoquants are *L-shaped*
  - There is *NO* substitution available between inputs
  - The output can be made with only a *specific* proportion of *K* and *L*
  - Cannot increase *q* unless increase both *K* and *L* in that specific proportion



A, B, and C represents technically efficient combinations of inputs

Adding more *L* alone does not increase *q*. Likewise for *K* alone

Only one combination of *L* and *K* can be used to produce each level of *q*

Substitution among inputs is impossible

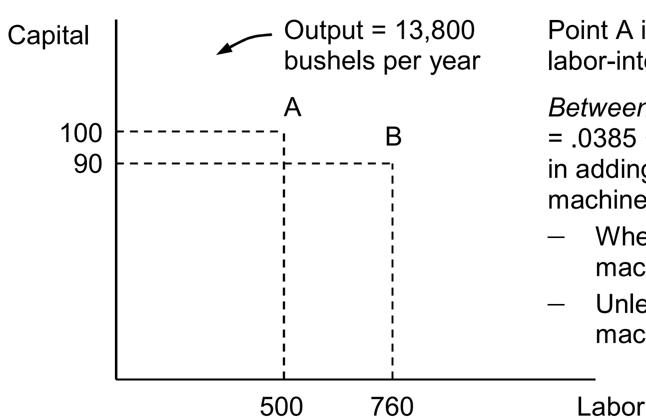
E.g., using jackhammers in the reconstruction of concrete sidewalk

A Production Function for Wheat: An Example

- Farmers can produce crops with different combinations of capital and labor
  - Crops in developed countries are grown with *capital-intensive* technology
  - Crops in developing countries are grown with *labor intensive* productions

(Why is it so?)

- Farm manager can use the *isoquant* to decide what combination of labor and capital will *maximize* profits from crop production
  - A: 500 hours of *L*, 100 units of *K*
  - B: *K* ↓ to 90, *L* must ↑ by 260 to 760 hours



Point A is more capital-intensive and B is more labor-intensive

Between A and B, MRTS = $-\Delta K / \Delta L = -(-10/260) = .0385 < 1$, reveals the nature of trade-off involved in adding labor and reducing the use of farm machinery

- When wage is equal to cost of running a farm machine, more capital ought to be used
- Unless labor is much less expensive than use of machine, production should be capital intensive

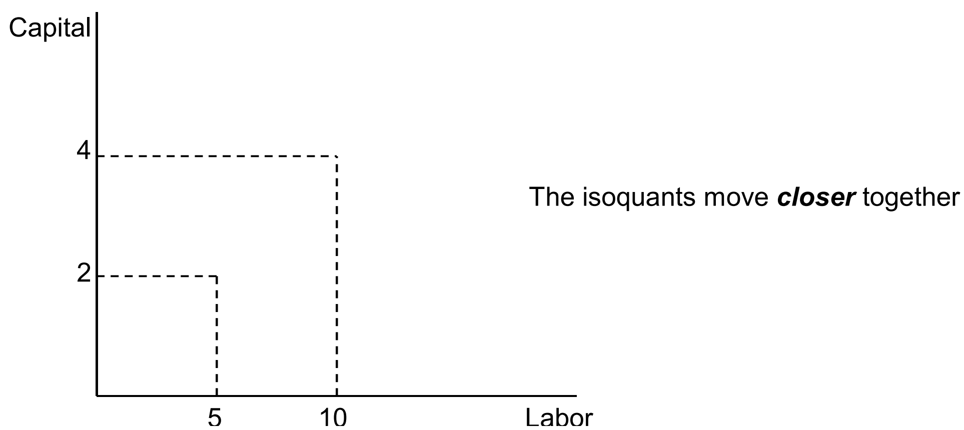
Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.

Returns to Scale

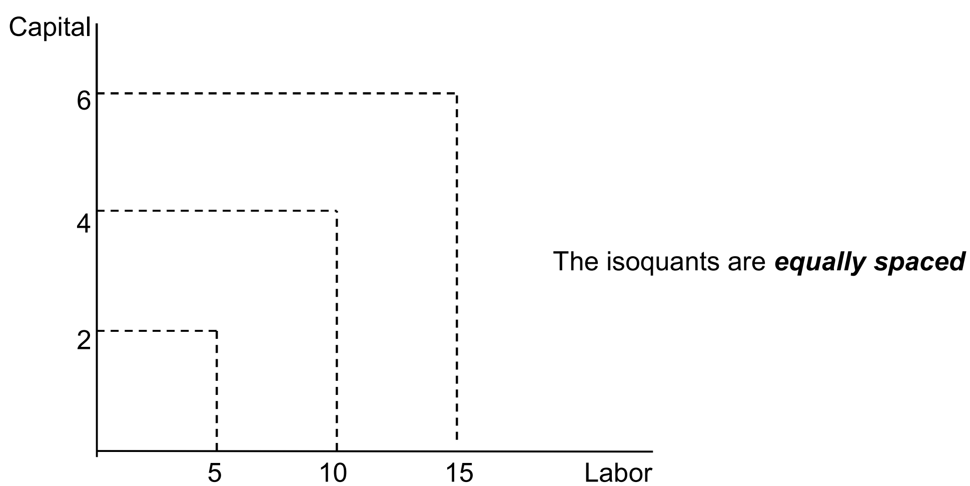
- How does a firm decide, in the long run, the best way to increase output?
  - *Change the scale of operation by increasing all inputs in proportion?*
  - *If inputs double, output will most likely increase but by how much?*

- Returns to scale refers to the relationship between *output* and *proportional increases* in all inputs
  - *Increasing* returns to scale $F(\lambda K, \lambda L) > \lambda F(K,L)$ where λ = proportional increase in inputs
  - *Constant* returns to scale $F(\lambda K, \lambda L) = \lambda F(K,L)$
  - *Decreasing* returns to scale $F(\lambda K, \lambda L) < \lambda F(K,L)$

- *Increasing* returns to scale: output *more than doubles* when all inputs are *doubled*
  - *Size* matters: *Larger* output associated with *lower* cost (e.g., cars)
  - *One* firm is more efficient than *many* (e.g., utilities)

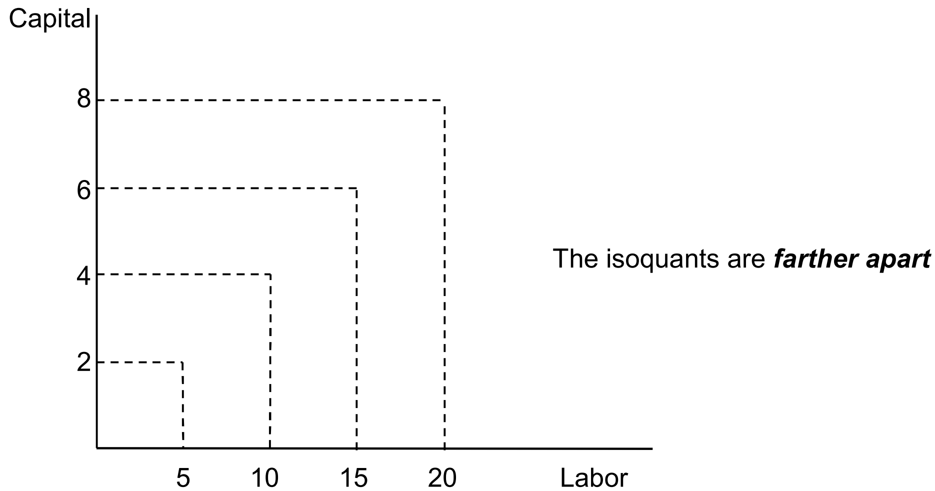


- *Constant* returns to scale: output *doubles* when all inputs are *doubled*
  - *Size* does not affect productivity (e.g., travel agencies)
  - May have *many* producers



Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.

- *Decreasing* returns to scale: output *less than doubles* when all inputs are *doubled*
  - *Decreasing* efficiency with *large* size
  - *Reduction* of entrepreneurial abilities



- Returns to scale vary across firms and industries
  - All else equal, the *greater* the returns to scale, the *larger* firms in an industry are likely to be
  - *Manufacturing* industries that are more *capital-intensive* are more likely to have increasing returns to scale than *service-oriented* industries that are more *labor-intensive*

- Most firms have production functions that exhibit *first increasing, then constant, and ultimately decreasing* returns to scale
  - When output levels are *low*, *doubled* all inputs may lead to *more than doubled* output based on an increase in the opportunity for *specialization*
  - As the firm grows, the opportunities for *specialization* may diminish and a *doubling* of all inputs will lead to only a *doubling* of output
  - At some point, the firm grows so large that output will be *less than doubled* when inputs are *doubled* because of management *diseconomies*

Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 6.

Quick Quiz

1. You are an employer seeking to fill a vacant position on an assembly line. Are you more concerned with the average product of labor or the marginal product of labor for the last person hired? If you observe that your average product is just beginning to decline, should you hire any more workers? What does this situation imply about the marginal product of your last worker hired?

2. Can an isoquant ever slope upwards? Explain.

3. For each of the following examples, draw a representative isoquants. What can you say about the marginal rate of technical substitution in each case?
   a. A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.
   b. A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.
   c. A firm requires exactly two full-time workers to operate each piece of machinery in the factory.

4. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor increased, and the other factor is held constant?
   a. $q = 3L + 2K$
   b. $q = (2L + 2K)^{1/2}$
   c. $q = 3LK^2$